Corporate Finance

Time Value of Money

Chapter 4

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Basic Definitions

- Present Value (PV)
  - The current value of future cash flows discounted at the appropriate discount rate.
  - Value at $t = 0$ on a time line.
- Future Value (FV)
  - The amount an investment is worth after one or more periods.
  - Future money on a time line.
- Interest rate (r)
  - Terminology depends on usage: Discount rate, Cost of capital, Opportunity cost of capital, Required return.

Time Line of Cash Flows

- Today is $t = 0$;
- $t = 1$ is the end of Period 1;
- $+CF$ is the Cash Inflow, $-CF$ is the Cash Outflow, PMT is the Constant CF.

Figure 1: Time Line of Cash Flows

Time Line of Cash Flows Cont'd

Example

Time line for a $100 lump sum due at the end of year 2:

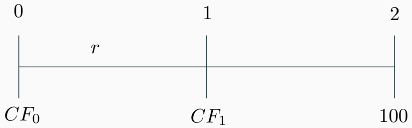

Figure 2: Time Line of Cash Flows

Future Values: General Formula

- Let FV be the future value, PV be the present value, r be the period interest rate expressed as a decimal, and t be the number of periods, then
$$FV = PV(1 + r)^t$$
- Future value interest factor is given by $(1 + r)^t$

Example

Suppose you invest \$100 for one year at 10% per year, then the future value in one year is $100(0.10) = 10$. Value in one year is the principal plus interest, that is, $100 + 10 = 110$. The future value is

$$FV = 100(1 + 0.10) = 110.$$

Suppose you leave the money in for another year. Then, you will have

$$FV = 100(1.10)(1.10) = 100(1.10)^2 = 121.00$$

two years from now.

Effects of Compounding

- Simple Interest
  - Interest earned only on the original principal.
- Compound Interest
  - Interest earned on principal and on interest received;
  - Interest on Interest is the interest earned on reinvestment of previous interest payments.

Example

Consider the previous example. FV with simple interest

$$FV = 100 + 10 + 10 = 120$$

FV with compound interest

$$FV = 100(1.10)^2 = 121$$

The extra \$1 comes from the interest of $0.10(10) = 1.00$ earned on the first interest payment.

Effects of Compounding Cont'd

Example

Suppose you invest the $100 from the previous example for 5 years. How much would you have?

$$\begin{aligned} FV &= PV(1+r)^t \\ &= 100(1.10)^5 \\ &= 100(1.6105) = 161.05 \end{aligned}$$

Year	Beginning Amount	Interest Earned	Ending Amount
1	100	10	110
2	110	11	121
3	121	12.10	133.10
4	133.10	13.31	146.41
5	146.41	14.64	161.05
		61.05	

Effects of Compounding Cont'd

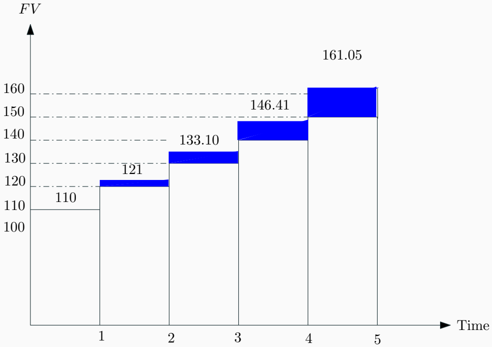

Figure 3: *FV*, Simple and Compound Interest

Excel Spreadsheet Functions

- Excel TVM Functions:
  - $FV(rate, nper, pmt, pv)$;
  - $PV(rate, nper, pmt, fv)$;
  - $RATE(nper, pmt, pv, fv)$;
  - $NPER(rate, pmt, pv, fv)$.
- Use the formula icon ($_x$) when you cannot remember the exact formula.
- Click on the Excel icon to open a spreadsheet containing four different examples.

Future Values

Example

Suppose you had a relative deposit \$10 at 5.5% interest 200 years ago. How much would the investment be worth today?

$$
\begin{aligned}
FV =& PV(1+r)^t \\
=& 10(1.055)^{200} \\
=& 10(44718.984) = 447,189.84
\end{aligned}
$$

Excel Solution:

$$FV(Rate, Nper, PMT, PV) = FV(0.055, 200, 0, -10) = 447,189.84$$

Future Value Cont'd: General Growth Formula

Example

Suppose your company expects to increase unit sales of widgets by 15% per year for the next 5 years. If you currently sell 3 million widgets in one year, how many widgets do you expect to sell in 5 years?

$$\begin{aligned} FV &= PV(1+r)^t \\ &= 3(1.15)^5 \\ &= 3(2.0114) = 6.0341 \end{aligned}$$

Excel Solution:

$$FV(Rate, Nper, PMT, PV) = FV(0.15, 5, 0, 3) = -6.0341$$

Future Value

- Important Relationship I
  - For a given interest rate, the longer the time period, the higher the future value;
  - For a given r, as t increases, FV increases.
- Important Relationship II
  - For a given time period, the higher the interest rate, the larger the future value;
  - For a given t, as r increases, FV increases.

Present Values

- The current value of future cash flows discounted at the appropriate discount rate.
- Value at t=0 on a time line.
- It is worth less than face value, because
    - Opportunity Cost
    - Risk & Uncertainty

$$Discount\ Rate = f(Time, Risk)$$

- Rearrange $FV = PV(1+r)^t$ to solve for PV:

$$PV = \frac{FV}{(1+r)^t} = FV(1+r)^{-t}$$

    - Discounting is defined as finding the PV of one or more future amounts.

Present Values Cont'd

Example

What is the PV of \$100 due in 3 Years if $r = 10\%$? Note that finding PVs is discounting, and it's the reverse of compounding.

Figure 4: FV, Simple and Compound Interest

$$\begin{aligned} PV =& FV(1+r)^{-t} \\ =& 100(1.10)^{-3} = 75.13 \end{aligned}$$

Excel Solution:

$$PV(0.10, 3, 0, 100) = -75.13$$

Present Values Cont'd

Example (Single Period)

Suppose you need $10,000 in one year for the down payment on a new car. If you can earn 7% annually, how much do you need to invest today?

$$\begin{aligned} PV =& FV(1+r)^{-t} \\ =& 10,000(1.07)^{-1} = 9,345.79 \end{aligned}$$

Excel Solution:

$$PV(0.07, 1, 0, 10,000) = -9,345.79$$

Present Values Cont'd

Example (Multi-Periods)

You want to begin saving for your daughter's college education and you estimate that she will need \$150,000 in 17 years. If you feel confident that you can earn 8% per year, how much do you need to invest today?

$$\begin{aligned} PV &= FV(1+r)^{-t} \\ &= 150,000(1.08)^{-17} = 40,540.34 \end{aligned}$$

Excel Solution:

$$PV(0.08, 17, 0, 150000) = -40,540.34$$

Present Values Cont'd

- Important Relationship I
  - For a given interest rate, the longer the time period, the lower the present value;
  - For a given r, as t increases, PV decreases.

Example

What is the present value of $500 to be received in 5 years? 10 years? The discount rate is 10%.

$$PV_5 = \frac{500}{(1+0.10)^5}$$
$$= -310.46$$
$$PV_{10} = \frac{500}{(1+0.10)^{10}}$$
$$= -192.77$$

Present Values Cont'd

- Important Relationship II
  - For a given time period, the higher the interest rate, the smaller the present value;
  - For a given t, as r increases, PV decreases.

Example

What is the present value of $500 received in 5 years if the interest rate is 10%? 15%?

$$\begin{aligned} PV &= \frac{500}{(1+0.10)^5} \\ &= -310.46 \end{aligned}$$

$$\begin{aligned} PV &= \frac{500}{(1+0.15)^5} \\ &= -248.59 \end{aligned}$$

Discount Rate

- To find the implied interest rate, rearrange the basic PV equation and solve for r:

$$FV = PV(1 + r)^t$$
$$r = \left(\frac{FV}{PV}\right)^{1/t} - 1$$

Example

You are looking at an investment that will pay \$1200 in 5 years if you invest \$1000 today. What is the implied rate of interest?

$$r = \left(\frac{1200}{1000}\right)^{1/5} - 1 = .03714 = 3.714\%$$

Excel Solution:

$$RATE(5, 0, -1000, 1200) = 0.03714$$

Finding the Number of Periods

- Start with basic equation $FV = PV(1+r)^t$ and solve for t:

$$t = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+r)}$$

- Excel Solution:

$$NPER(Rate, Pmt, PV, FV)$$

Example

You want to purchase a new car and you are willing to pay $20,000. If you can invest at 10% per year and you currently have $15,000, how long will it be before you have enough money to pay cash for the car?

$$\begin{aligned} \frac{FV}{PV} &= \frac{20,000}{15,000} = 1.333 \\ \ln(1.333) &= 0.2877 \\ \ln(1.10) &= 0.0953 \\ t &= \frac{0.2877}{0.0953} = 3.0189 \end{aligned}$$